Hough Transform Clarification

Initial equation for our constrained ellipse:

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

Derivative of the above equation in terms of x:

$$\frac{2(x-x_0)}{a^2} + \frac{2(y-y_0)}{b^2}(\frac{dy}{dx}) = 0$$

This derivative gives us two substitutions, as follows:

$$x = x_0 - \frac{a^2}{b^2}(y-y_0)(\frac{dy}{dx})$$

$$y = y_0 - \frac{b^2}{a^2}(x-x_0)(\frac{dy}{dx})^{-1}$$

Applying these two substitutions to our initial equation, we get the following solutions:

$$y = y_0 \pm \frac{b}{\sqrt{\frac{a^2}{b^2}(\frac{dy}{dx})^2 + 1}}$$

$$x = x_0 \pm \frac{a}{\sqrt{\frac{b^2}{a^2}(\frac{dy}{dx})^{-2} + 1}}$$

Finally, if we make the substitution $\tan\phi = \frac{dy}{dx}$, our solutions become:

$$y = y_0 \pm \frac{b}{\sqrt{\frac{a^2(\tan\phi)^2}{b^2} + 1}}$$

$$x = x_0 \pm \frac{a}{\sqrt{\frac{b^2}{a^2(\tan\phi)^2} + 1}}$$