

## Key for 2nd Midterm Exam

1. *15 points*

Use resolution to prove that  $(S \vee R)$  is entailed by the following set of propositional expressions:

1.  $\neg(\neg Q) \wedge Z$
2.  $\neg W$
3.  $(\neg W \wedge Q) \Rightarrow (\neg P)$
4.  $(W \wedge Z) \Rightarrow S$
5.  $Q \Rightarrow (S \vee P)$
6.  $(P \wedge Q) \Rightarrow R$

**Answer:** we convert to CNF and add the negated goal

1a.  $Q$

1b.  $Z$

2.  $\neg W$

3.  $W \vee \neg Q \vee \neg P$

4.  $\neg W \vee \neg Z \vee S$

5.  $\neg Q \vee S \vee P$

6.  $\neg P \vee \neg Q \vee R$

$\neg Ga.$   $\neg S$

$\neg Gb.$   $\neg R$

We resolve  $\neg Gb.$  with 6. and obtain

7.  $\neg P \vee \neg Q$

then 7. with 5.

8.  $\neg Q \vee S$

and 8. with 1a.

9.  $S$

and, finally, 9. with  $\neg Ga$  to obtain the empty clause.

2. 20 points

(a) Write the following sentences in predicate calculus:

1. Every student has taken at least one computer science course.  
 $\forall x Student(x) \Rightarrow \exists y CScourse(y) \wedge Take(x, y)$
2. A student has taken at most one computer science course.  
 $\exists x Student(x) \wedge [\forall y CScourse(y) \wedge Take(x, y) \Rightarrow [\forall z CScourse(z) \wedge z \neq y \Rightarrow \neg Take(x, z)]]$   
or, equivalently,  
 $\exists x Student(x) \wedge [\forall y CScourse(y) \wedge Take(x, y) \Rightarrow [\forall z CScourse(z) \wedge Take(x, z) \Rightarrow z = y]]$
3. Every student has been in every building on campus.  
 $\forall x \forall y Student(x) \wedge Building(y) \Rightarrow Visited(x, y)$
4. There is a student who has been in every room of at least one building on campus.  
 $\exists x Student(x) \wedge \exists y Building(y) \wedge [\forall z Room(z, y) \Rightarrow Visited(x, z)]$
5. Every student has been in at least one room of every building on campus.  
 $\forall x \forall y Student(x) \wedge Building(y) \Rightarrow \exists z Room(z, y) \wedge Visited(x, z)$

(b) transform the expressions you wrote in part (a) to CNF.

- 1a  $\neg Student(x) \vee CScourse(S(x))$
- 1b  $\neg Student(x) \vee Take(a, S(x))$
- 2  $Student(S0) \wedge \neg CScourse(y) \vee \neg Take(S0, y) \vee \neg CScourse(z) \vee z = y \vee \neg Take(S0, z)$
- 3  $\neg Student(x) \vee \neg Building(y) \vee Visited(x, y)$
- 4a  $Student(S1)$
- 4b  $Building(S2)$
- 4c  $\neg Room(z, S2) \vee Visited(S1, z)$
- 5a  $\neg Student(x) \vee \neg Building(y) \vee Room(S3, y)$
- 5b  $\neg Student(x) \vee \neg Building(y) \vee Visited(x, S3)$

(c) Does 5 entail 3? If yes, prove it by resolution (adding additional expressions if needed). If not, explain why not.

**Answer:** to answer we need to negate 3, add it to a knowledge base which contains 5 and any other expression we would have to add, and prove a contradiction.

Given 3.  $\forall x \forall y \text{ Student}(x) \wedge \text{Building}(y) \Rightarrow \text{Visited}(x, y)$

we negate it and obtain

3a  $\text{Student}(S4)$

3b  $\text{Building}(S5)$

3c  $\neg \text{Visited}(S4, S5)$

We need to add a new expression

6.  $\forall x \forall y \forall z \text{ Room}(z, y) \wedge \text{Visited}(x, z) \Rightarrow \text{Visited}(x, y)$

which in CNF is

6  $\neg \text{Room}(z, y) \vee \neg \text{Visited}(x, z) \vee \text{Visited}(x, y)$

We resolve 5a with 3a unifying  $\{x/S4\}$  and obtain

7  $\neg \text{Building}(y) \vee \text{Room}(S3, y)$

7 is resolved with 6 using unification  $\{z/S3\}$  and we obtain

8  $\neg \text{Visited}(x, S3) \vee \text{Visited}(x, y) \vee \neg \text{Building}(y)$

which is resolved against 3c with unification  $\{x/S4, y/S5\}$  to obtain

9  $\neg \text{Visited}(S4, S3) \vee \neg \text{Building}(S5)$

which we resolve with 5b unifying  $\{x/S4\}$  to obtain

10  $\neg \text{Student}(S4) \neg \text{Building}(S5)$

We resolve it with 3a to obtain

11  $\neg \text{Building}(S5)$

which, finally, resolved with 3b produces

12 NIL

### 3. 10 points

Specify if each of the following expressions represents correctly the corresponding English statement. If not explain why not and correct it.

1. Every cat owner loves all animals.

$\forall x \forall z [\exists y \text{ Cat}(y) \Rightarrow \text{Owns}(x, y)] \Rightarrow \text{Loves}(x, z)$

**Answer:** The sentence says if someone owns a cat, that person loves all things. We need  $\wedge$  instead of  $\Rightarrow$  and we need to specify that  $z$  is an animal. The correct sentence is:

$\forall x \forall z [\exists y \text{ Cat}(y) \wedge \text{Owns}(x, y) \wedge \text{Animal}(z)] \Rightarrow \text{Loves}(x, z)$

2. No person would harm a cat.

$\forall x \forall y \text{ Person}(x) \wedge \text{Cat}(y) \wedge \neg \text{Harm}(x, y)$

**Answer:** we need  $\Rightarrow$  instead of  $\wedge$   
 $\forall x \forall y \text{ Person}(x) \wedge \text{Cat}(y) \Rightarrow \neg \text{Harm}(x, y)$

4. 15 points

You are given the following pairs of clauses where upper case letters indicate constants, lower case letters indicate variables, functions, or predicates. Consider each pair independently of the others. In each pair variables with the same name are meant to be the same variable. For each of the pairs specify if they can be resolved. If yes show the results of the unification process, if not explain why.

1.  $p(B, C, x, z, f(A, z, B))$  and  $\neg p(y, z, y, C, w)$

**Answer:**  $\{y/B, z/C, x/B, w/f(A, C, B)\}$

2.  $r(f(y), y, x)$  and  $\neg r(x, f(A), f(v))$

**Answer:**  $\{x/f(y), y/f(A), v/f(A)\}$

3.  $q(f(A, x), x)$  and  $\neg q(f(z, f(z, D)), z)$

**Answer:**  $\{z/A, x/f(A, D), \text{FAIL}\}$

5. 15 points

Show the backed-up up values for all the nodes in the following game tree and show the branches that are pruned by alpha-beta. For each branch pruned, explain briefly why alpha-beta can prune it. Follow the convention used in the textbook to examine the branches in the tree from left to right.

