
Algorithms and Data Structures

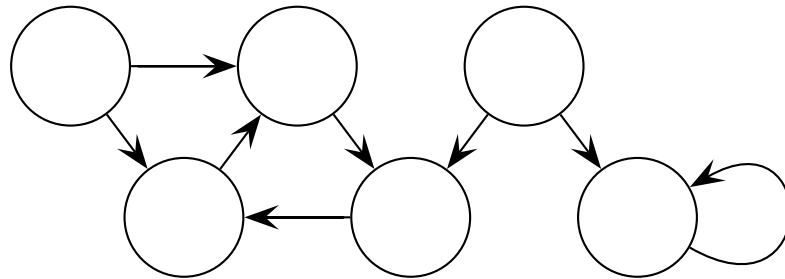
CSCI 4041

Session 23

Depth-First Search

dfs(Graph G):

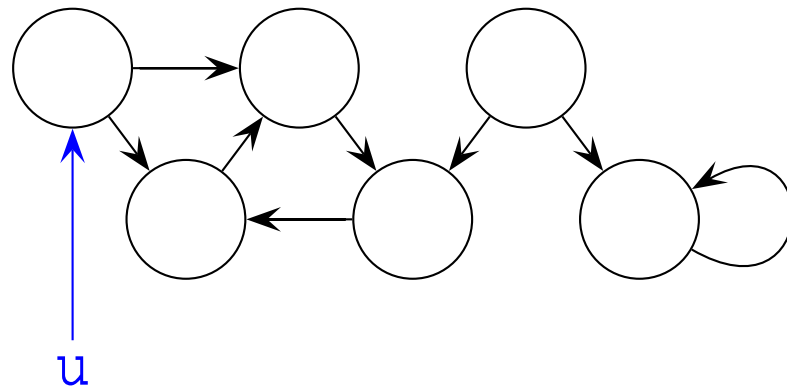
```
for each Vertex  $u \in \text{vertices}(G)$   
     $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
    if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (1)

dfs(Graph G):

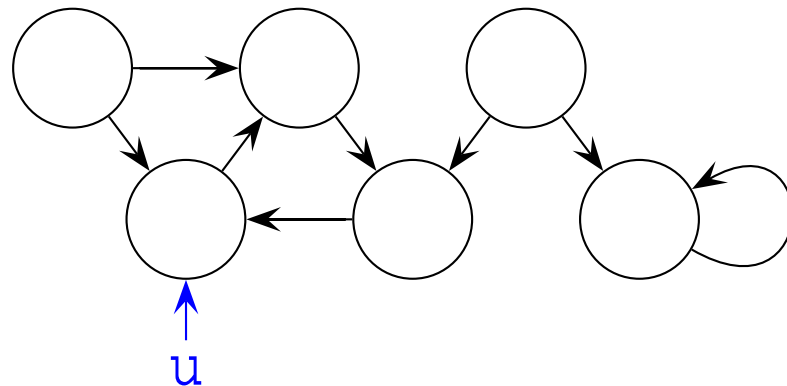
```
for each Vertex  $u \in \text{vertices}(G)$   
     $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
    if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (2)

dfs(Graph G):

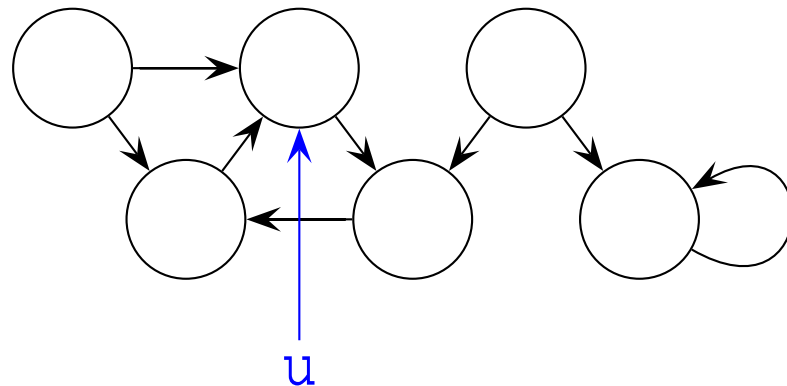
```
for each Vertex  $u \in \text{vertices}(G)$   
     $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
    if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (3)

dfs(Graph G):

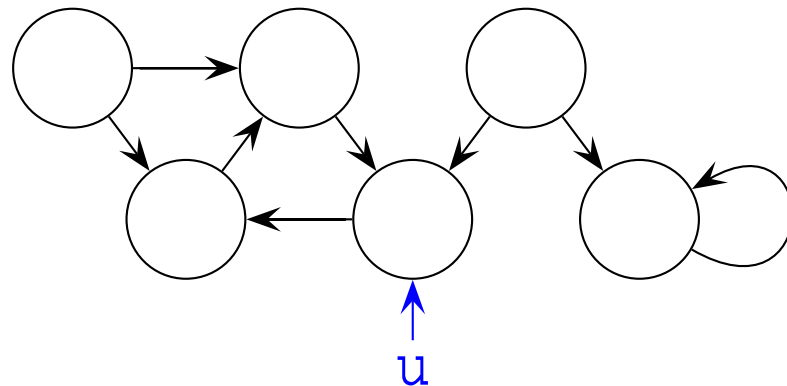
```
for each Vertex  $u \in \text{vertices}(G)$   
     $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
    if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (4)

dfs(Graph G):

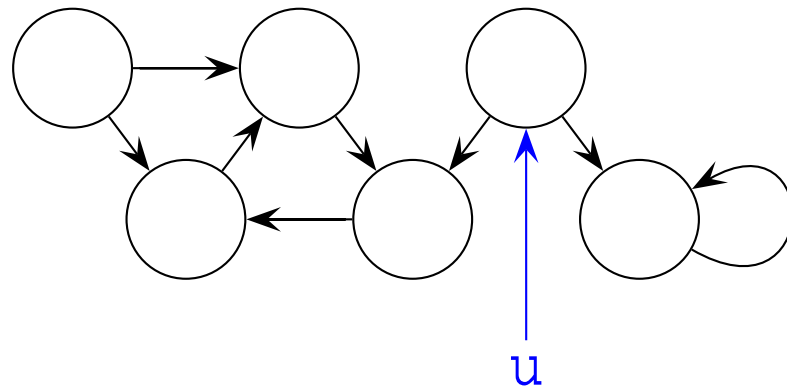
```
for each Vertex  $u \in \text{vertices}(G)$   
     $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
    if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (5)

dfs(Graph G):

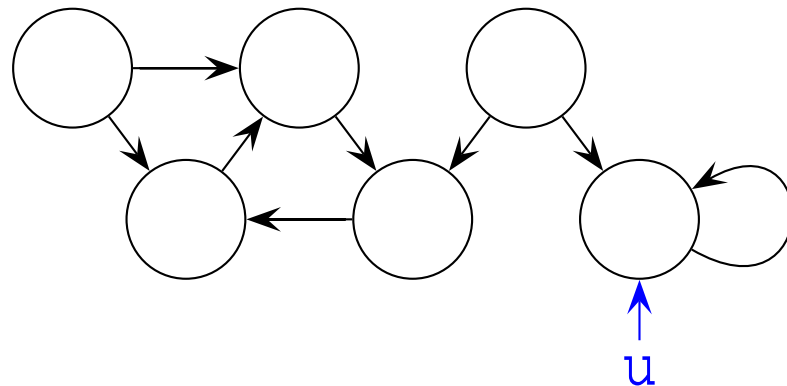
```
for each Vertex  $u \in \text{vertices}(G)$   
     $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
    if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (6)

dfs(Graph G):

```
for each Vertex  $u \in \text{vertices}(G)$   
     $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
    if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (7)

dfs(Graph G):

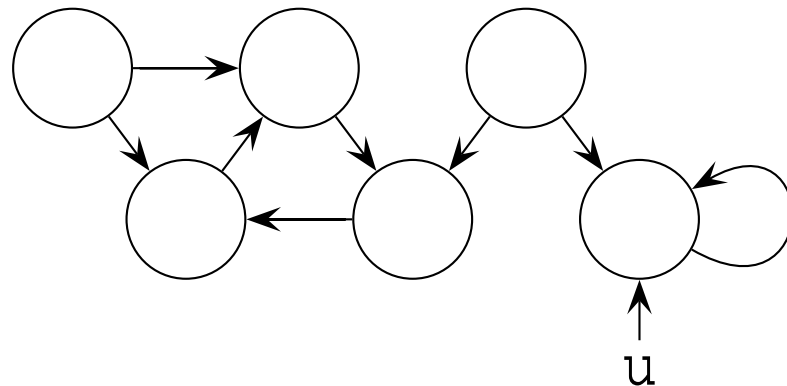
for each Vertex $u \in \text{vertices}(G)$

$\text{col}(u) \leftarrow \text{WHITE}$, $p(u) \leftarrow \text{NIL}$

$\text{time} \leftarrow 0$

for each Vertex $u \in \text{vertices}(G)$

if $\text{col}(u) = \text{WHITE}$ **then** $\text{dfsVisit}(u)$



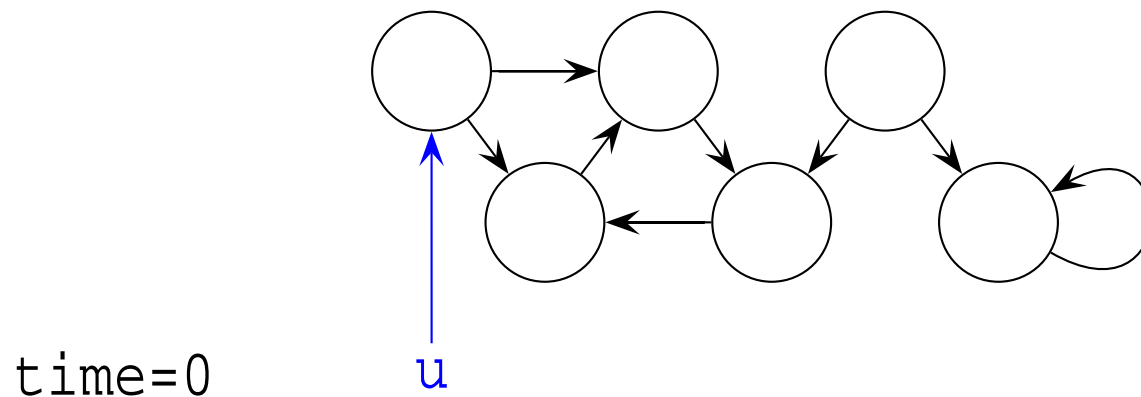
$\text{time} = 0$

Depth-First Search (8)

dfs(Graph G):

```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```

dfsVisit(Vertex u)



DFS (8) | DFS Visit

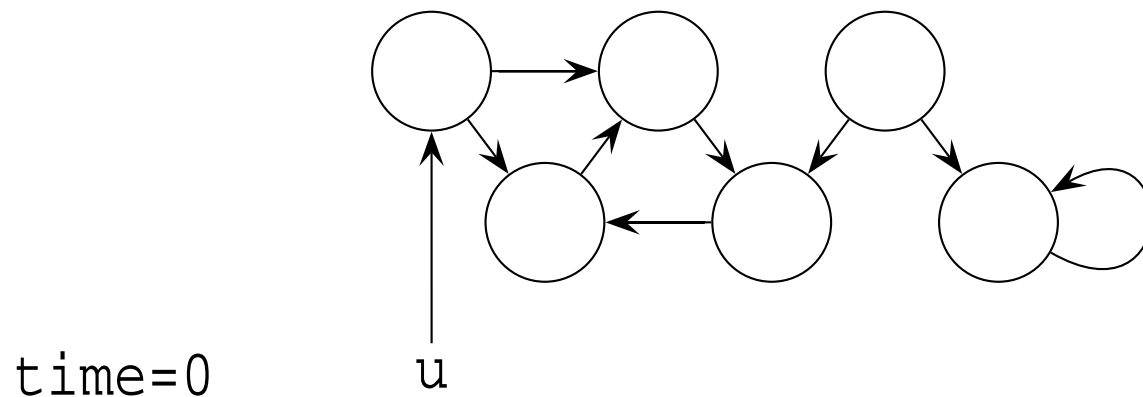
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



DFS (8) | DFS Visit (1)

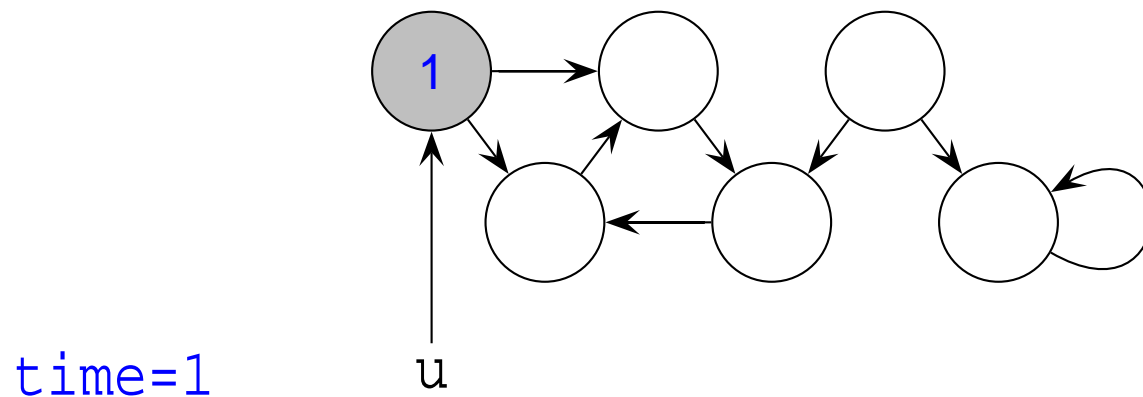
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



DFS (8) | DFS Visit (2)

dfsVisit(Vertex u):

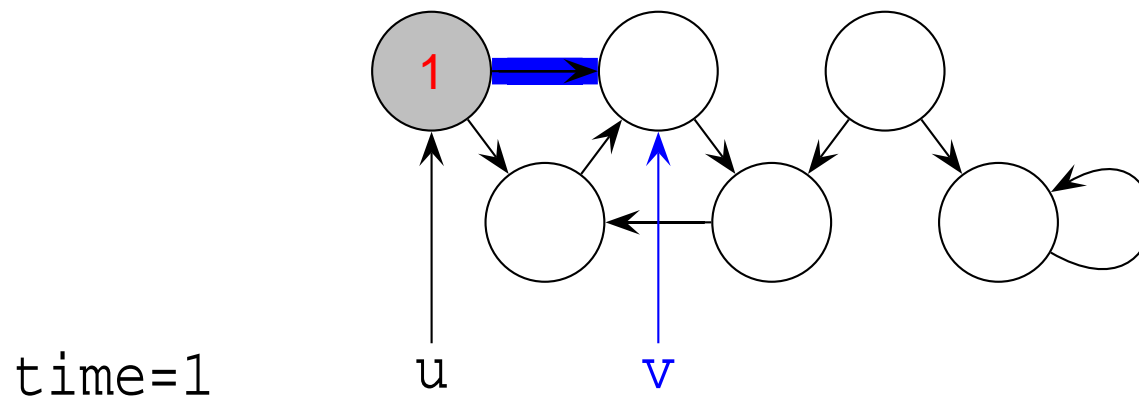
col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time

dfsVisit(Vertex u)



DFS (8) | DFS Visit (2) | DFS Visit

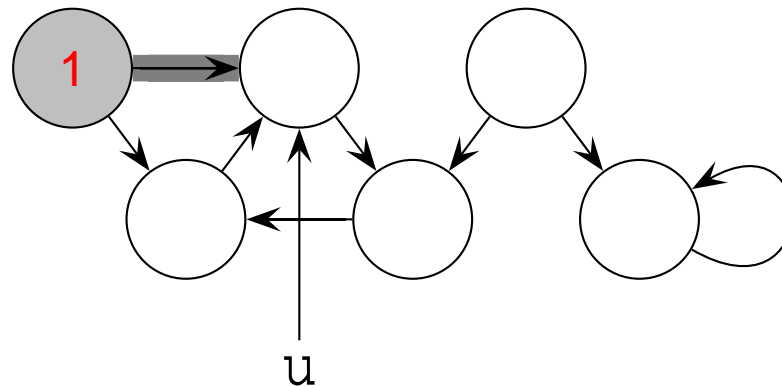
dfsVisit(Vertex u):

col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time



DFS (8) | DFS Visit (2) | DFS Visit (1)

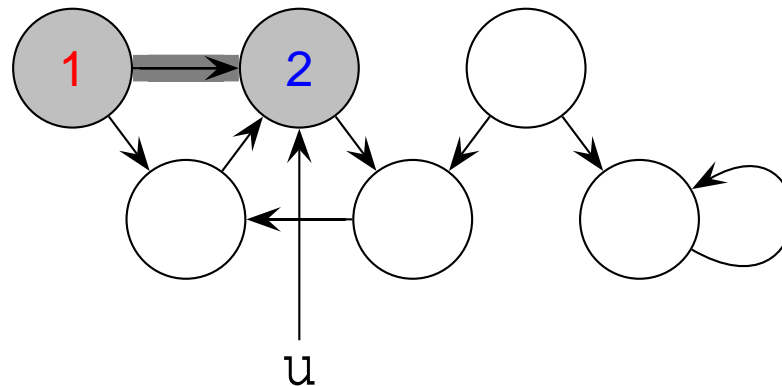
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



DFS (8) | DFS Visit (2) | DFS Visit (2)

dfsVisit(Vertex u):

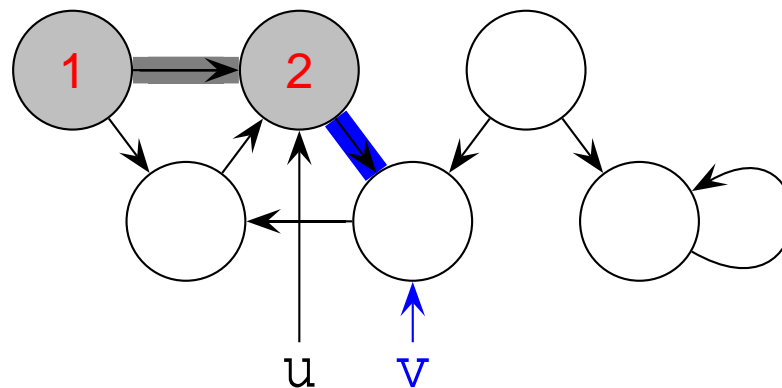
col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time

dfsVisit(Vertex u)



time=2

DFS (8) | DFS Visit (2) | DFS Visit (2) | DFS Visit

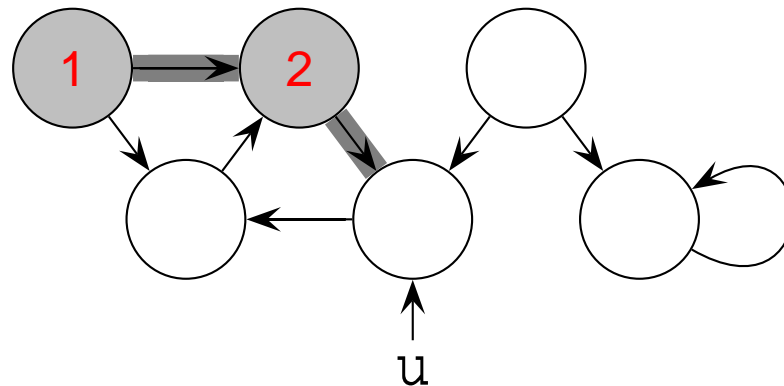
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



time=2

DFS (8) | DFS Visit (2) | DFS Visit (2) | DFS Visit (1)

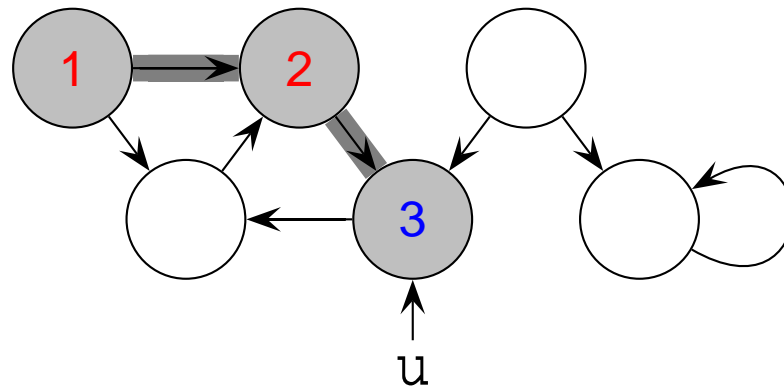
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



$\text{time}=3$

DFS (8) | DFS Visit (2) | DFS Visit (2) | DFS Visit (2) | DFS Visit (1)

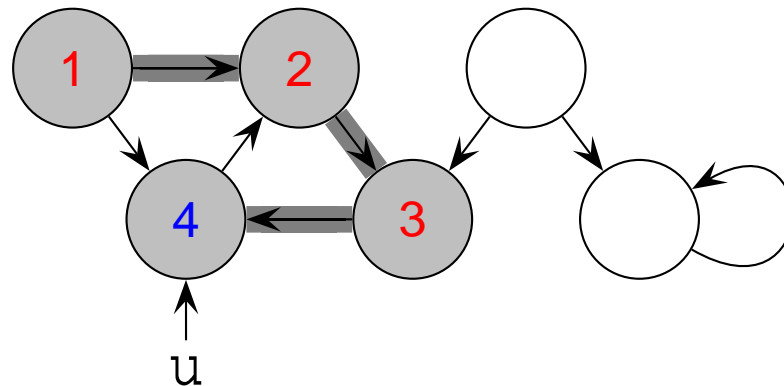
dfsVisit(Vertex u):

col(u) ← GRAY, time ← time+1, d(u) ← time

for each Vertex v ∈ adjacent(u)

if col(v)=WHITE **then** p(v) ← u, dfsVisit(v)

col(u) ← BLACK, time ← time+1, f(u) ← time



time=4

DFS (8) | DFS Visit (2) | DFS Visit (2) | DFS Visit (2) | DFS Visit (2)

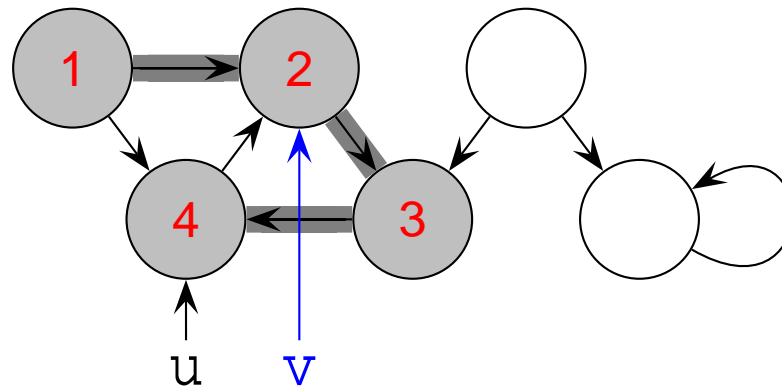
dfsVisit(Vertex u):

col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time



time=4

DFS (8) | DFS Visit (2) | DFS Visit (2) | DFS Visit (2) | DFS Visit (done)

dfsVisit(Vertex u):

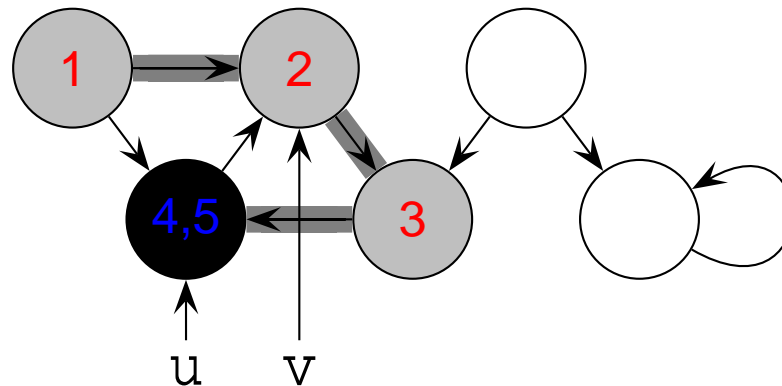
col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time

time=5



DFS (8) | DFS Visit (2) | DFS Visit (2) | DFS Visit (2)

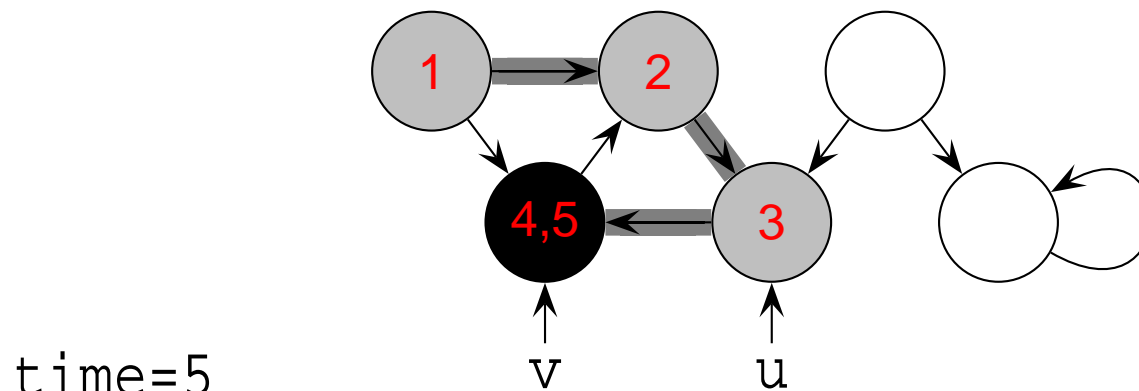
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



DFS (8) | DFS Visit (2) | DFS Visit (2)

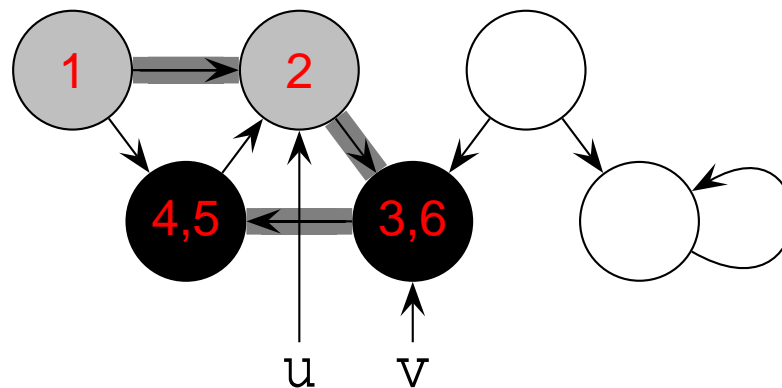
dfsVisit(Vertex u):

col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time



time=6

DFS (8) | DFS Visit (2) | DFS Visit (done)

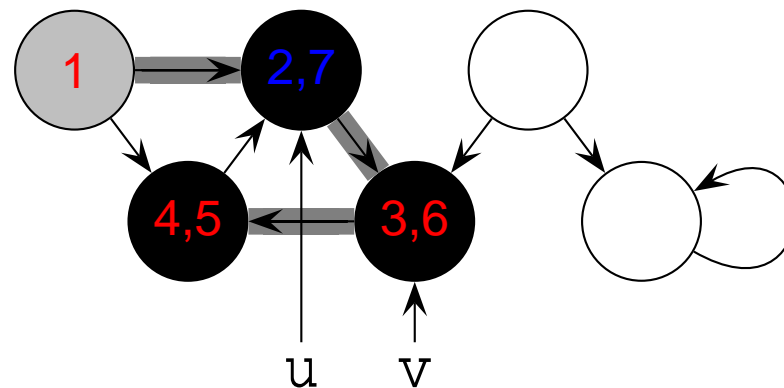
dfsVisit(Vertex u):

col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time



DFS (8) | DFS Visit (2)

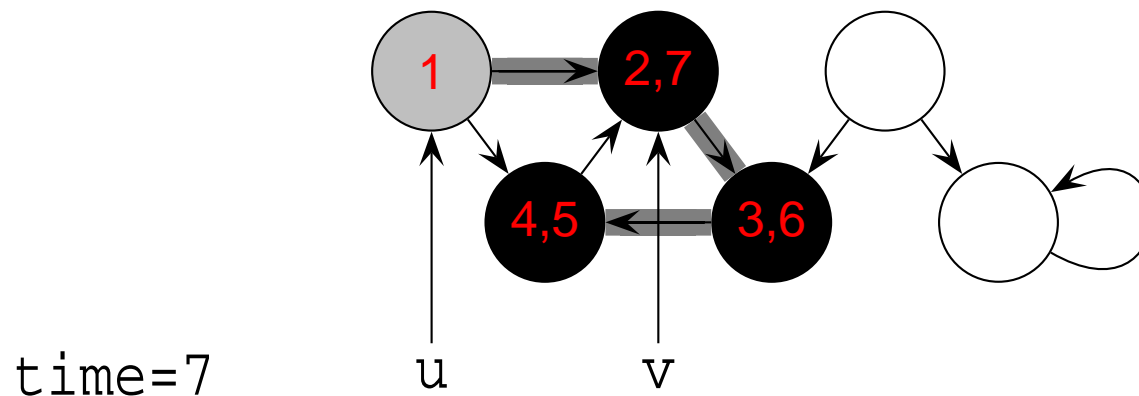
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}$, $\text{time} \leftarrow \text{time}+1$, $d(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $p(v) \leftarrow u$, $\text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}$, $\text{time} \leftarrow \text{time}+1$, $f(u) \leftarrow \text{time}$



DFS (8) | DFS Visit (3)

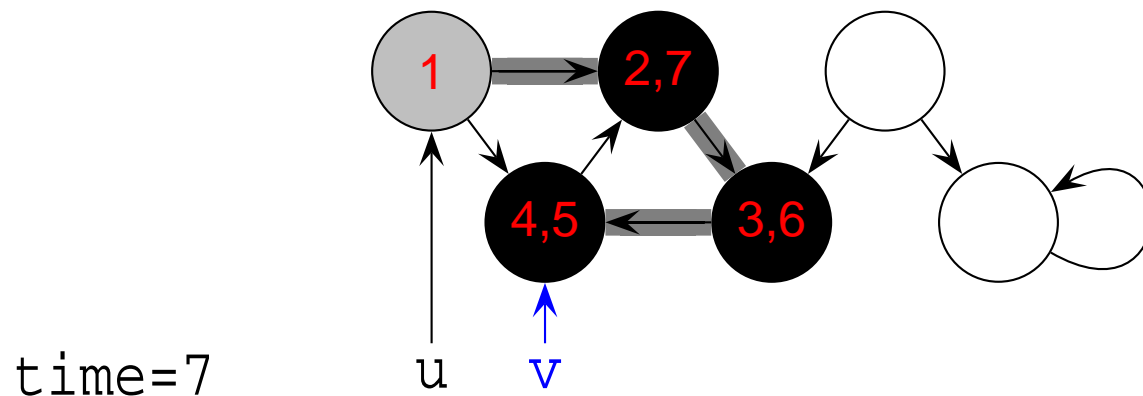
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

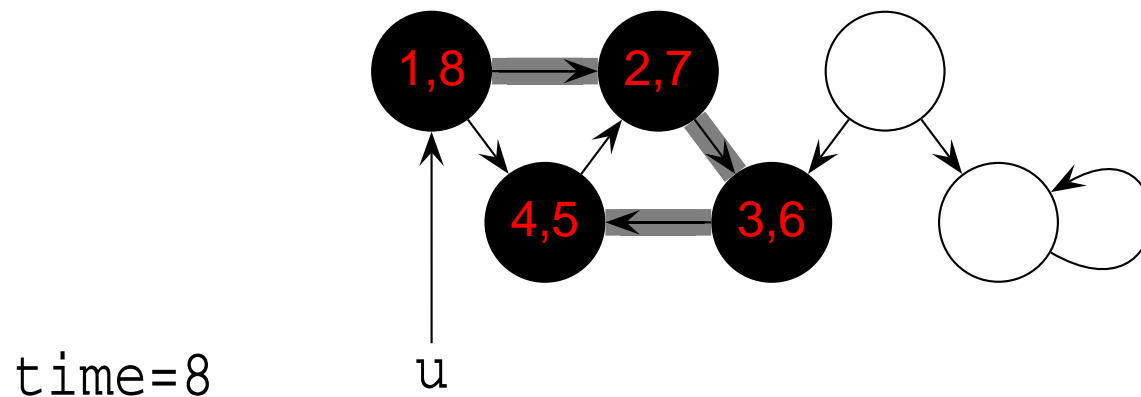
$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



Depth-First Search (8)

dfs(Graph G):

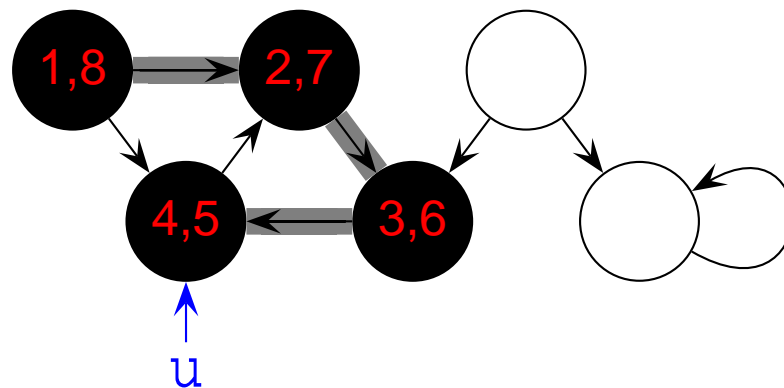
```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (9)

dfs(Graph G):

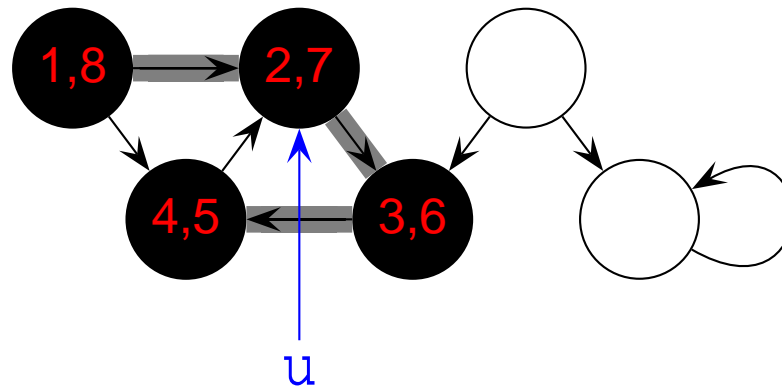
```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (10)

dfs(Graph G):

```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```

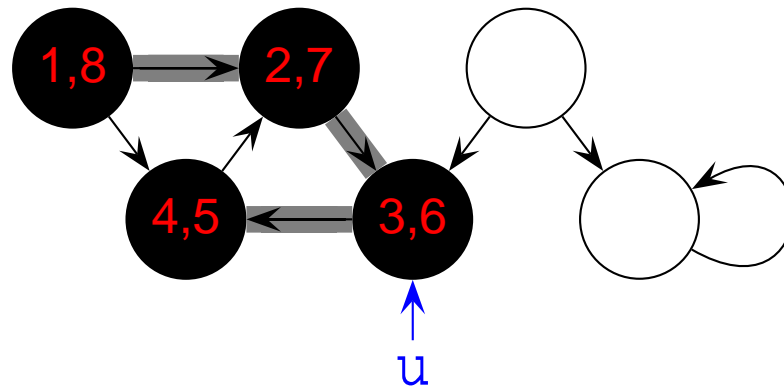


time=8

Depth-First Search (11)

dfs(Graph G):

```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



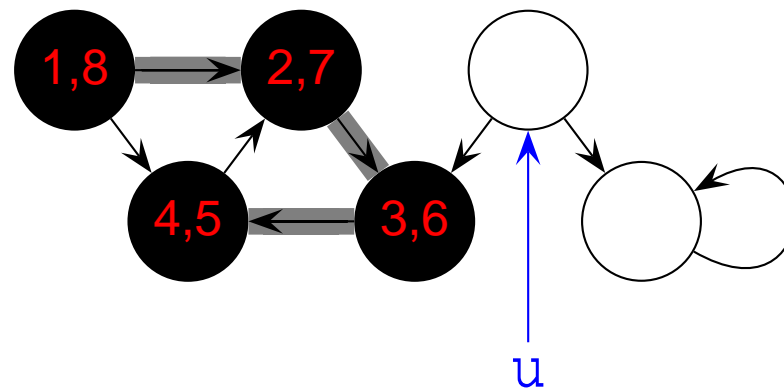
time=8

Depth-First Search (12)

dfs(Graph G):

```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```

dfsVisit(Vertex u)



DFS (12) | DFS Visit

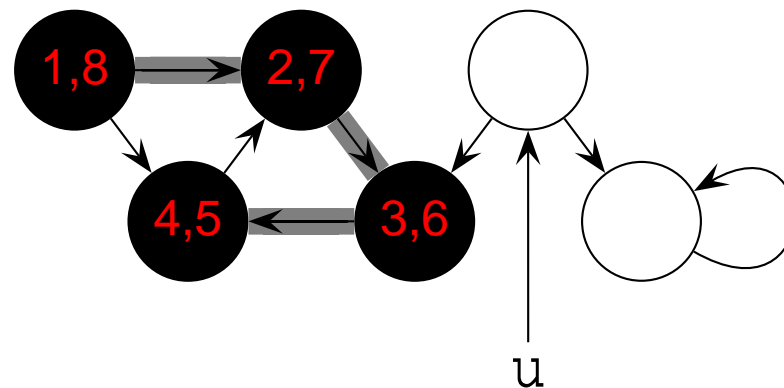
dfsVisit(Vertex u):

col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time



DFS (12) | DFS Visit (1)

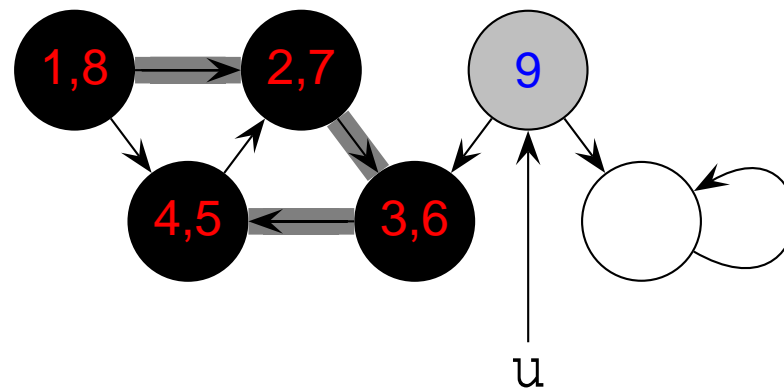
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



$\text{time} = 9$

DFS (12) | DFS Visit (2)

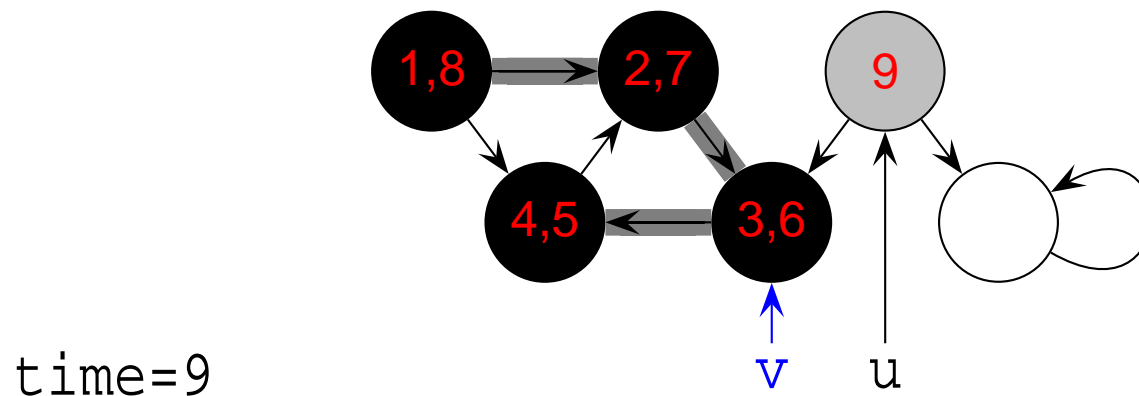
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



DFS (12) | DFS Visit (3)

dfsVisit(Vertex u):

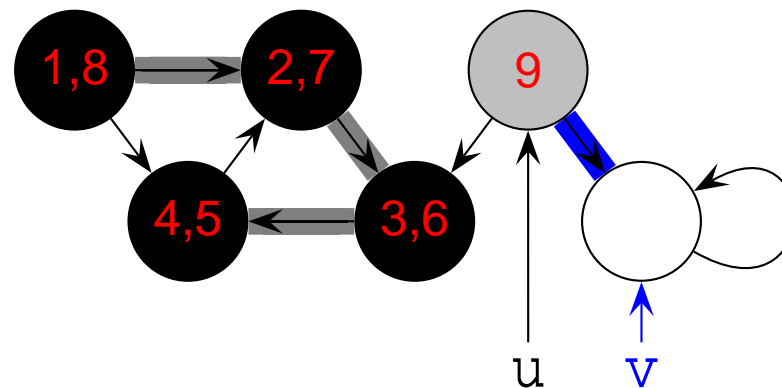
col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time

dfsVisit(Vertex u)



DFS (12) | DFS Visit (3) | DFS Visit

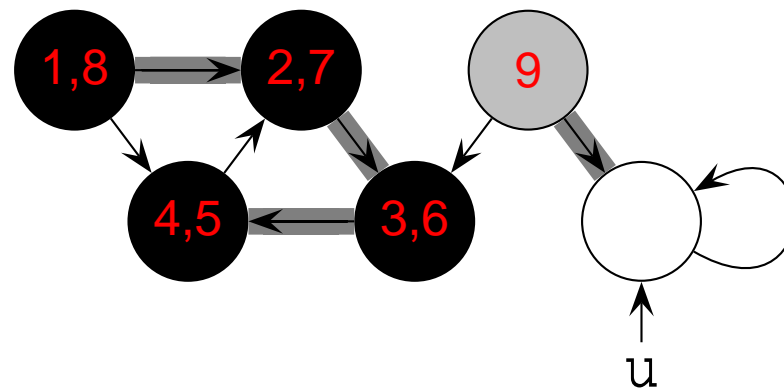
dfsVisit(Vertex u):

col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time



time=9

DFS (12) | DFS Visit (3) | DFS Visit (1)

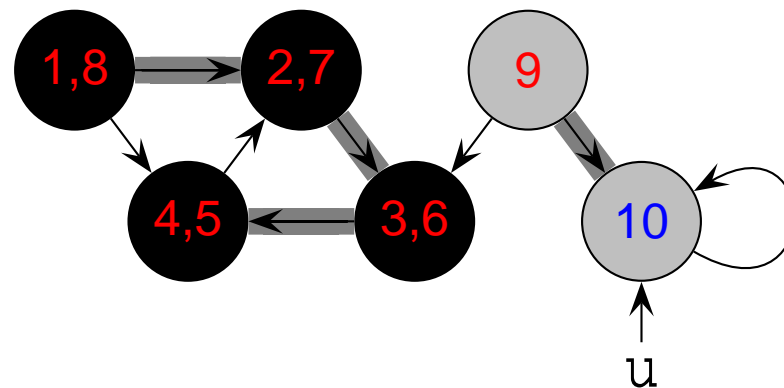
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



$\text{time}=10$

DFS (12) | DFS Visit (3) | DFS Visit (2)

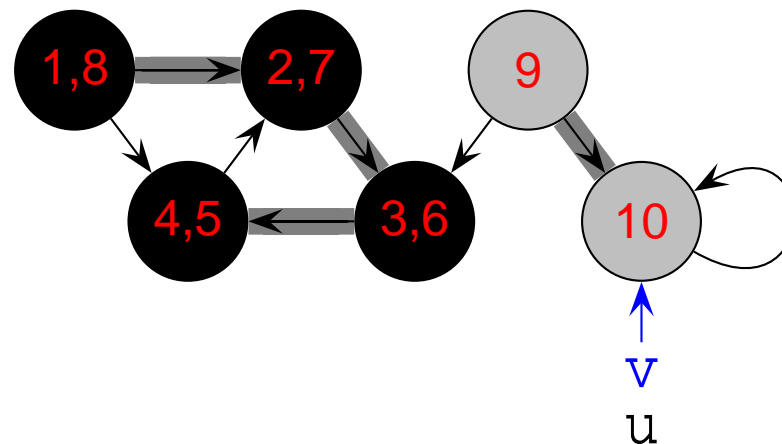
dfsVisit(Vertex u):

col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time



time=10

DFS (12) | DFS Visit (3) | DFS Visit (done)

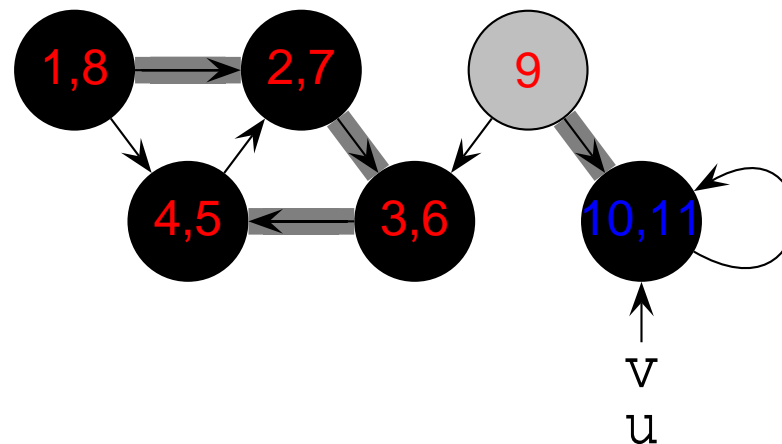
dfsVisit(Vertex u):

col(u) \leftarrow GRAY, time \leftarrow time+1, d(u) \leftarrow time

for each Vertex v \in adjacent(u)

if col(v)=WHITE **then** p(v) \leftarrow u, dfsVisit(v)

col(u) \leftarrow BLACK, time \leftarrow time+1, f(u) \leftarrow time



time=11

DFS (12) | DFS Visit (3)

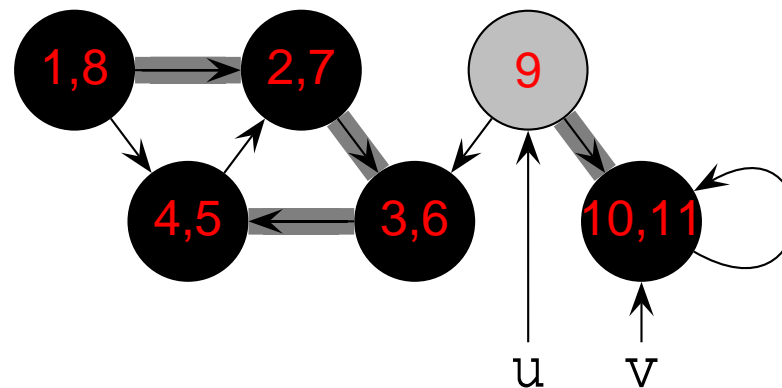
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$



DFS (12) | DFS Visit (done)

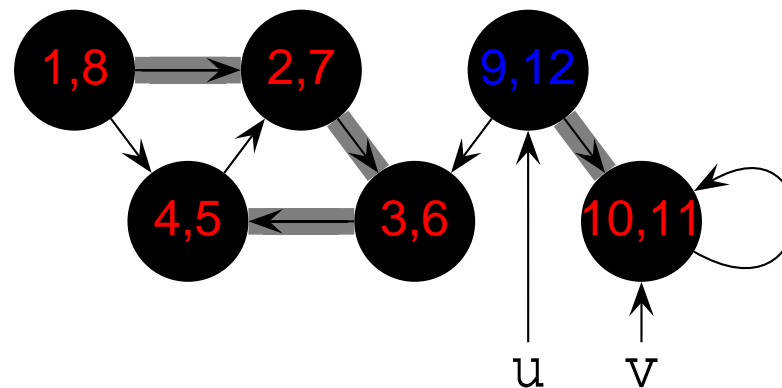
dfsVisit(Vertex u):

$\text{col}(u) \leftarrow \text{GRAY}, \text{time} \leftarrow \text{time}+1, \text{d}(u) \leftarrow \text{time}$

for each Vertex $v \in \text{adjacent}(u)$

if $\text{col}(v) = \text{WHITE}$ **then** $\text{p}(v) \leftarrow u, \text{dfsVisit}(v)$

$\text{col}(u) \leftarrow \text{BLACK}, \text{time} \leftarrow \text{time}+1, \text{f}(u) \leftarrow \text{time}$

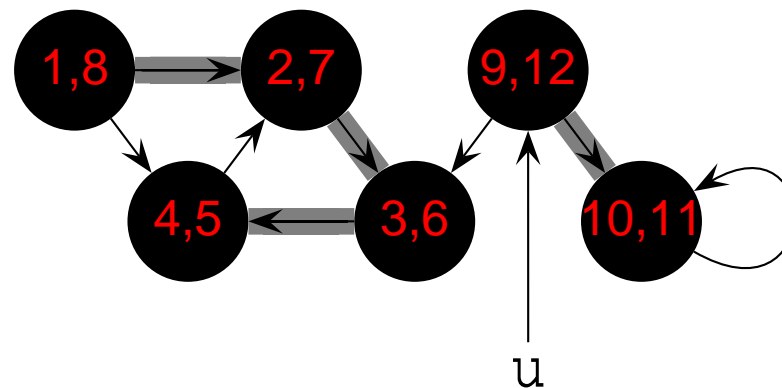


$\text{time} = 12$

Depth-First Search (12)

dfs(Graph G):

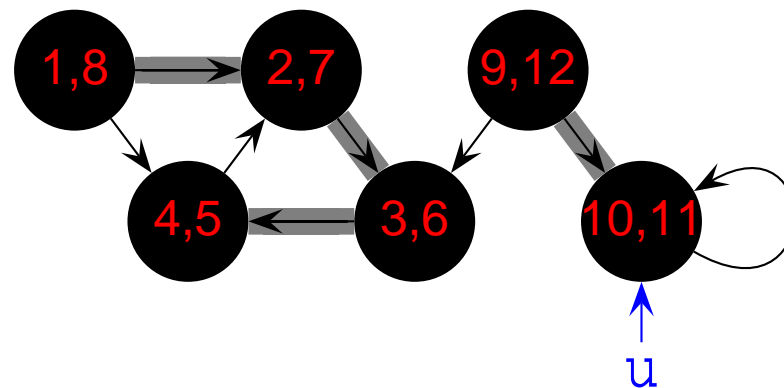
```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



Depth-First Search (13)

dfs(Graph G):

```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```

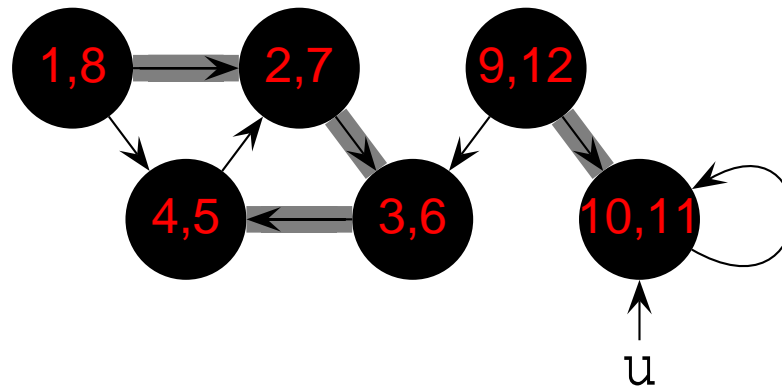


time=12

Depth-First Search (done)

dfs(Graph G):

```
for each Vertex  $u \in \text{vertices}(G)$   
   $\text{col}(u) \leftarrow \text{WHITE}$ ,  $p(u) \leftarrow \text{NIL}$   
 $\text{time} \leftarrow 0$   
for each Vertex  $u \in \text{vertices}(G)$   
  if  $\text{col}(u) = \text{WHITE}$  then  $\text{dfsVisit}(u)$ 
```



time=12